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# FORMULATION OF DESIGN ENVELOPE CRITERION IN TERMS OF DETERMINISTIC SPECTRAL PROCEDURE

by

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## FORMULATION OF DESIGN ENVELOPE CRITERION IN TERMS OF DETERMINISTIC SPECTRAL PROCEDURE

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SUMMARY

An existing Design Envelope approach to meeting aircraft limit-load requirements for flight in continuous turbulence, using power-spectral methods, is reformulated in a manner which makes no distinction between linear and nonlinear aircraft response. Computational techniques for implementing the new procedure in applications to nonlinear aircraft are discussed and compared with existing simulation methods.



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LIST OF CONTENTS

## 1 INTRODUCTION

Mandatory aircraft limit-load requirements for flight in continuous turbulence are generally met by using power spectral procedures to compute the loads. Two approaches are in general use. One is a Design Envelope approach and the other is Mission Analysis. In the Design Envelope approach, a response factor $\bar{A}$ is calculated and multiplied by a specified gust intensity $U_\sigma$ to obtain the design load for a series of points throughout the design envelope. In Mission Analysis, mission profiles are analysed in order to obtain probabilities of exceeding various load levels and a design probability is specified, from which design loads may be found.

With the advent of advanced technology aircraft employing active controls, interest has turned to the problem of demonstrating compliance with the requirements in the situation where the overall transfer function of aircraft plus control system may be nonlinear. A particular source of nonlinearity is that due to control saturation (in amplitude or rate). As the power-spectral-density (PSD) method, as usually formulated, applies specifically to the situation where the aircraft transfer function is linear, the question has arisen as to the most rational and convenient way to extend the formulation to apply to nonlinear aircraft response.

In this paper we consider specifically the Design Envelope approach and demonstrate that it can be reformulated in a manner which makes no distinction between linear and nonlinear response, thus providing a rational basis for its extension to the nonlinear aircraft. The reformulation depends upon the demonstrated result[1,2] that, whilst power-spectral procedures for evaluating the stochastic response of linear systems are generally implemented by means of frequency-plane calculations, an alternative, but exactly equivalent, time-plane method exists in the form of a worst-case analysis in which the maximum response to a class of deterministic inputs subject to a prescribed constraint is found. This is shown below to lead naturally to a Deterministic Spectral Procedure (DSP) for implementing Design Envelopment requirements, equally applicable to linear and nonlinear aircraft.

## 2 EQUIVALENT DETERMINISTIC ANALYSIS

The starting point is a result, due to Papoulis[3], concerning the maximum response of a linear system to a deterministic input subject to a prescribed constraint. The system frequency-response function will be denoted by $H_y(i\omega)$ and a deterministic input by $x(t)$ , or its associated Fourier transform $X(\omega)$ .

The prescribed constraint on the input takes the form

$$\pi \int_0^\infty | X(\omega) |^2 G(\omega) \, d\omega \leqslant 1 \, , \tag{1}$$

where $G(\omega)$ is an arbitrary positive function, $G(\omega) \geqslant 0$ . It can be demonstrated that the weak inequality in equation (1) may be replaced by an equality without altering any of the subsequent results. However, for the present purpose of extending the method to apply to nonlinear systems, the constraint as expressed in equation (1) has advantages.

By an application of the Schwarz inequality, Papoulis[3] showed that the maximum amplitude $\max_x | y(t) |$ of system response $y(t)$ , when the deterministic input $x(t)$ is varied over the class of functions subject to the constraint of equation (1), is given by

$$\left\{ \max_x \, | y(t) | \right\}^2 = \frac{1}{\pi} \int_0^\infty \frac{|H_y(i\omega)|^2}{G(\omega)} \, d\omega \, . \tag{2}$$

This result may be used[1,2] to provide a bridge between a system analysis based on a deterministic worst-case search on the one hand, and standard power-spectral methods to determine the dynamic response of linear systems to stochastic inputs on the other. The usual basis[4] for the latter approach is the equation

$$\sigma_y^2 = \int_0^\infty | H_y(i\omega) |^2 \, \phi(\omega) \, d\omega \, , \tag{3}$$

for the variance of the response variable $y(t)$ , having frequency-response function $H_y(i\omega)$ , due to a stationary stochastic input with power spectrum $\phi(\omega)$ . That is, $\sigma_y^2$ is evaluated as an integral in the frequency plane. However, by relating $G(\omega)$ to the inverse of the power spectral density of the input:

$$G(\omega) = \left\{ 2\pi\phi(\omega) \right\}^{-1} \, , \tag{4}$$

it follows from equations (2) and (3) that

$$\sigma_y^2 = \left\{ \max_x \, | y(t) | \right\}^2 / 2 \, . \tag{5}$$

Thus the variance of the stochastic response may be obtained by evaluating the maximum response resulting from a deterministic worst-case analysis. In this analysis, the maximum response $\max_x | y(t) |$ is found with respect to a class of deterministic inputs $x(t)$ subject to the constraint

$$\frac{1}{2}\int_0^\infty \frac{|X(\omega)|^2}{\phi(\omega)}\, d\omega \leq 1 \ , \tag{6}$$

(from equations (1) and (4)).

## 3 DETERMINISTIC SPECTRAL PROCEDURE

In the standard Design Envelope approach to limit loads[5,6], which assumes a linear dynamic model for the aircraft, a design load $y_d$ is calculated using the equation

$$y_d = \bar{A}\, U_\sigma \ , \tag{7}$$

where $\bar{A}$ is an aircraft-dependent dynamic response factor, calculated using PSD theory as the ratio of standard deviations of output and input:

$$\bar{A} = \frac{\sigma_y}{\sigma(\text{gust})} \ . \tag{8}$$

In equation (8), $\sigma_y$ is calculated in the frequency plane, using equation (3), and $U_\sigma$ is a turbulence intensity which is prescribed in the requirements[5,6] and depends on altitude and aircraft speed.

As pointed out explicitly in Ref 7, equations (7) and (8), combined with equations (5) and (6), lead to the result

$$y_d = \max_{u,t} | y(t) | \ , \tag{9}$$

where the maximum aircraft response is evaluated with respect to a deterministic family of gust inputs $u(t)$, subject to the constraint

$$\| u(t) \| \leq U_\sigma \ . \tag{10}$$

Here the 'norm' of $u(t)$ is defined by the equation

$$\| u(t) \|^2 = \int_0^\infty \frac{|U(\omega)|^2}{\phi(\omega)} d\omega , \tag{11}$$

(and $U(\omega)$ is the Fourier transform of $u(t)$ ). Equations (9) to (11) above form the basis of the proposed Deterministic Spectral Procedure.

Whereas equation (7), the usual basis for Design Envelope calculations, is only applicable as it stands to linear aircraft dynamic models, equations (9) to (11), although derived above specifically for the linear problem, contain no reference to linearity and are applicable equally when nonlinearities are present. Indeed the inequality (10) is in the spirit of a 'Design Envelope', in that a requirement formulated in these terms would extend the Envelope comprising altitudes and speeds to encompass a specific family of gusts $u(t)$, namely those satisfying (10), to which the aircraft must be exposed without exceeding its design load at each altitude and speed condition.

## 4 IMPLEMENTATION

The calculation of $y_d$ on the basis of equations (9) to (11) requires that a sequence of deterministic samples $u(t)$ be generated and that the maximum response $\max_u | y(t) |$ be found subject to the constraint imposed by equation (10). There are many ways of solving this problem of constrained optimisation, from which a user should be free to choose. Here we simply make some general points about implementation, underline some possible pitfalls, and point to means of avoiding them.

First, it should be noted that the form of the power spectrum $\phi(\omega)$ prescribed in power-spectral requirements[5,6] is such that the samples $u(t)$ are continuous (otherwise inequality (10) is violated) and can without loss of generality be initialized at value zero at some arbitrary starting time. Moreover, the sample $u^*(t)$ which maximises the response, equation (9), will have a duration determined by the response times of the aircraft modes; for linear systems, $\{u^*(t)\}_{lin}$ is related in a known analytical manner[1-3] to the impulse-response function of the aircraft dynamics. This not only provides a guide as to the required duration of the samples $u(t)$ but also, in instances where a related linear system can be constructed, provides a possible starting point for the search for $u^*(t)$ when the dynamics are nonlinear.

To implement the search for $u^*(t)$, and hence $y_d$, equation (9), $u(t)$ will typically be parameterized using a discrete set $a_1, a_2, .., a_n, ...$ of real

coefficients which define the coordinates of the space within which the constraint, equation (10), is imposed and the multi-dimensional search performed.

In practice, it will not be feasible to perform an exhaustive search of this multi-dimensional space, owing to the number of coefficients $a_i$ involved. Thus some form of directed search will be required. Examples of such search techniques applied to turbulence time histories are described in Ref 8. However, one consequence of system nonlinearity is that, unlike the situation for linear systems, systematic 'hill-climbing' procedures can converge to a local, rather than the required global, maximum. Not only must the user satisfy himself that this problem has been overcome, but he will be required to satisfy the certification authority that this is so. To meet the latter requirement, we propose that a combination of systematic hill-climbing and randomisation be employed, and the results be displayed in a format that exhibits not just the computed value of $\max_u |y(t)|$ but the entire history of the simulation process from which this maximum value was deduced. There is, in fact, a substantial literature concerning closely analogous problems in statistical physics[9,10] which shows how the steps of a simulation may efficiently combine a systematic search for a 'worst case', which significantly reduces simulation time, with an element of randomisation which prevents the search halting at subsidiary local, rather than global, maxima. Such a technique has been termed[9] 'importance sampling' in that it involves the generation of random samples and yet weights the choice of inputs according to their importance in causing large response values.

To be more specific, it is proposed that such a simulation study be performed in two phases. In Phase 1, the coefficients $a_i$ are generated purely randomly, resulting in a set of random time histories $u(t)$. For example, the sequence $a_i$ can be chosen as successive values in a 'white noise' time history, and a sample $u(t)$ obtained by passing this white noise through a filter which shapes the output to have approximately a von Karman spectrum. Alternatively, a similar result can be achieved by choosing[8,11] the $a_i$ to be coefficients of a prescribed set of deterministic functions. Whilst this 'random' Phase of the study bears some resemblance to currently-used simulation methods for analysing the gust response of aircraft with nonlinear dynamics[7], there are significant differences. In contrast to existing techniques, for each sample $u(t)$ thus generated, or 'run', only the single largest response value $\max_t |y(t)|$ is recorded. Samples $u(t)$ which violate equation (10) are rejected (or modified, see below). As the simulation proceeds, a histogram of these maximum values is formed and an associated exceedance plot generated.

The amplitudes of the coefficients $a_i$ should be chosen so that the condition

$$\| u(t) \| = U_\sigma \quad , \qquad (12)$$

is satisfied in a proportion of the runs. This can be achieved by making an initial choice of amplitude in which equation (10) is *violated* in a number of the samples generated, and subsequently scaling down each such sample $u(t)$ until equation (12) is satisfied exactly, to form a modified input which is actually used in the 'run'.

In Phase 2 of the proposed simulation study, the more systematic procedures described earlier, incorporating directed search and 'importance sampling', are introduced. This Phase will systematically bias the choice of samples $u(t)$ so as to extend the tail of the measured exceedance distribution for $\max_t | y(t) |$ towards the desired asymptote $y = y_d$ (equation (9)). It is envisaged that, in practice, some prescribed safety factor would be introduced to allow for the gap between the tail of the measured exceedance curve and the position of the true asymptote.

## 5 COMPARISON WITH EXISTING SIMULATION METHODS

An existing alternative method[7,12] for extending continuous turbulence Design Envelope analysis from linear to nonlinear aircraft relates the design load defined by equation (7) to the rate-of-exceedance of response to a random Gaussian input with von Karman spectrum and intensity related to $U_\sigma$. The load that occurs with this specified rate-of-exceedance is found by numerical simulation using as inputs computer-generated time histories. This method, if applied to a linear aircraft model, gives the same results as a conventional frequency plane analysis and satisfies[7] a principle of equivalent safety, in that the design load for a nonlinear model will be exceeded at the same rate as that for a linear or linearized reference model.

It remains a subject for future work to compare, for specific nonlinear aircraft models, the design loads obtained by the above method[7] with those obtained by the proposed Deterministic Spectral Procedure. Here we make just a few general comments.

In the proposed procedure, in which inputs are constrained to lie entirely within the 'Design Envelope' specified by equations (10) and (11), the design

load $y_d$ is approached, but never actually reached. Conversely, it follows that, in the simulation methods of[7,12], measured responses at the level $y = y_d$ will in fact be associated with gust inputs that lie *outside* the Design Envelope. There is a unique gust time history from within the envelope which will exactly produce the design load ( and which for *linear* systems can be found analytically[1,2]). However, the probability of generating this time history in a random simulation is zero.

The two approaches differ, then, in that the methods of[7,12] exercise the aircraft at (and beyond) design-load level, but with inputs which lie outside the Design Envelope, whereas the proposed procedure restricts inputs to within, or on the boundary of, the envelope and thus approximates the design load from below. If the exceedance curve behaves smoothly in the vicinity of the design load, however, it is anticipated that the two approaches will give essentially equivalent results.

## 6 CONCLUSION

We repeat a comment from a recent paper[13] in which analogous factors are discussed in the context of the SDG method and nonlinear systems. Despite the apparently computationally intensive effort required to search for the maximum values of functions (as in equation (9)) in a multi-dimensional space, we claim that computational effort is being used in a more economical manner than in a standard[7,12] Monte Carlo simulation, in the sense that a systematic computer search to find the input pattern causing the maximum response is more efficient than simply waiting for patterns close to the critical input pattern to come up at random. To obtain adequate definition of the statistics (rate-of-exceedance) of system response at any particular threshold (particularly $y = y_d$ ) it is necessary to exercise the system with a set of input patterns which is sufficiently representative of the class of inputs which would actually cause the system response to approach that threshold during flight in real turbulence.

For linear aircraft, the existing Design Envelope method has been shown to define an envelope of gust inputs, equations (10) and (11), such that the design load is the maximum response that can be generated by this family of gusts. If a simulation is performed, as proposed in this paper, in which inputs are chosen entirely from within, or on the boundary of, this envelope the design load will be approached, but never reached, as the associated 'tuned gust' input will never be reproduced exactly. In the alternative simulation methods[7,12], in which the aircraft is exercised up to and beyond the design level, it follows that the gust inputs which actually produce these high loads are in fact 'off tune' gusts from

outside the Design Envelope. It is claimed[12] that smooth exceedance curves, which pass through the design load, can be produced in this way with less computational effort than would be expended in a directed search for the worst-case response associated with a specified envelope. This is not, in our view, evidence that such simulation methods are either more economical or more reliable as a means of estimating design loads.

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17. Abstract


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